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CUMULUS MODELLING USING THE MESOSCALE MODEL

BY

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§1 INTRODUCTION

So far the Mesoscale model has been verified against dry sea breeze predictions for Florida and the British Isles. For more realistic modelling of fronts and associated problems it is necessary to include moisture and convection processes in the model. As a start towards this problem, a scheme has been developed (based on the ideas of Liu and Orville 1969, Miller and Pearce 1974) to model the water phases and cloud microphysics. The model is currently being tested to simulate cumulonimbus convection.

The basic formulation follows that of Tapp and White (1976) with a non-hydrostatic compressible model in which the terms responsible for sound waves are treated implicitly. A four component system is considered consisting of air, water vapour, cloud water and rainwater. Because of the introduction of water variables, separate continuity equations to describe their density variation in space and time must be included in the equations. The cloud physics is modelled by defining the structure of the source/sink terms in these equations based on the microphysical processes between cloud rain and vapour.

This technical note describes the basic mathematical formulation of the model and discusses some numerical aspects that have arisen in solving the equations. The computer programs have been written in a general way so that the grid configuration may be changed easily, subject to certain computer hardware limitations.

§2 DYNAMICAL EQUATIONS

For definitions and notations used in this and subsequent sections refer to Appendix 1.

A four component system is considered consisting of air, water vapour, cloud and rainwater. It becomes a difficult mathematical task to consider the dynamics of each subsystem separately, especially if no prior knowledge of the average velocities involved is assumed. By making certain assumptions about the velocities, it is possible to reduce the problem to solving one set of momentum equations describing the bulk motion of the system as a whole. As a working hypothesis, the average velocities of air, water-vapour and cloud water are assumed identical and

equal to $\underset{\sim}{V}$. Rainwater is assumed to have a terminal fall velocity ($\underset{\sim}{V}_T$) relative to the other components. Therefore the rainwater velocity $\underset{\sim}{V}_r$ is defined as

$$\underset{\sim}{V}_r = \underset{\sim}{V} + \underset{\sim}{V}_T .$$

Given this information, the equations of motion for the system cannot predict the behaviour of $\underset{\sim}{V}$ without further knowledge of $\underset{\sim}{V}_T$. The rainwater fall velocity must be parameterised as a function of known variables in the system, the usual functional representation being

$$\underset{\sim}{V}_T = \underset{\sim}{V}_T(r);$$

that is, $\underset{\sim}{V}_T$ depends only on the rainwater concentration term r.

CONTINUITY EQUATION

Each component of the system will satisfy a certain continuity equation which governs its density variation in space and time. For dry air this is simply

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underset{\sim}{V}) = 0. \qquad (1)$$

For component μ ($\mu = v, c, r$) with sources/sinks S_μ, the appropriate equation is

$$\frac{\partial \rho_\mu}{\partial t} + \nabla \cdot (\rho_\mu \underset{\sim}{V}_\mu) = 0. \qquad (2)$$

Since the density of dry air appears on the right hand side of equation (2) the term S_μ is defined as

kgs (of component μ) per kg (of dry air) per sec.

Clearly with no other sources or sinks in the system

$$\sum_\mu S_\mu = 0.$$

Writing $\sigma_\mu = \frac{\rho_\mu}{\rho}$, then from equations (1) and (2) there results

$$\left(\frac{D}{Dt}\right)_\mu \sigma_\mu + \frac{\sigma_\mu}{\rho} \nabla \cdot \{\rho (\underset{\sim}{V}_\mu - \underset{\sim}{V})\} , \qquad (3)$$

where

$$\left(\frac{D}{Dt}\right)_\mu = \frac{\partial}{\partial t} + \underset{\sim}{V}_\mu \cdot \nabla ,$$

is the Eulerian derivative following the motion of each component. Note that for most conditions met in practice, the quantity σ_μ is typically of the order 10^{-3} kgs/kg.

MOMENTUM EQUATIONS

To simplify the problem, as mentioned earlier, only the equations of motion for the system as a whole will be derived. Considering a volume (τ), Newton's

second law of motion can be applied to give

$$\frac{\partial}{\partial t}\int_{\tau}\left\{\rho \underset{\sim}{v}+\sum_{\mu}\rho_{\mu}\underset{\sim}{v}_{\mu}\right\}d\tau$$

$$=-\int_{S}\left\{\underset{\approx}{P}_{d}+\sum_{\mu}\underset{\approx}{P}_{\mu}\right\}\cdot\hat{\underset{\sim}{n}}\,dS+\int_{\tau}\underset{\sim}{F}\left(\rho+\sum_{\mu}\rho_{\mu}\right)d\tau. \qquad (4)$$

S is the surface enclosing τ and $\hat{\underset{\sim}{n}}$ is a unit vector in the direction of the outward drawn normal to S. $\underset{\approx}{P}_{d}$ and $\underset{\approx}{P}_{\mu}$ are the pressure tensors for dry air and component μ respectively. $\underset{\sim}{F}$ is the body force acting on the system per unit mass (usually $-g\hat{\underset{\sim}{k}}$). For meteorological situations the pressure tensor can be separated into two terms representing the mass flow effect due to the average velocity and the thermal or random effects. Hence $\underset{\approx}{P}_{d}$ and $\underset{\approx}{P}_{\mu}$ may be written as

$$\underset{\approx}{P}_{d}=\rho\,\underset{\sim}{v}\underset{\sim}{v}+\underset{\approx}{P}, \qquad (5)$$

and

$$\underset{\approx}{P}_{\mu}=\rho_{\mu}\underset{\sim}{v}_{\mu}\underset{\sim}{v}_{\mu}+\underset{\approx}{P}_{\mu}.$$

For any quantity ϕ_{μ}, $\phi_{\mu}\underset{\sim}{v}_{\mu}$ can be regarded as the flow vector for that quantity. In this way the second velocity moment $\underset{\sim}{v}_{\mu}\underset{\sim}{v}_{\mu}$ times mass represents an average momentum flow for component μ - a tensor of second rank. The analysis is simplified if the total thermal pressure in the system is assumed hydrostatic. The tensors $\underset{\approx}{P}$ and $\underset{\approx}{P}_{\mu}$ become diagonal so that

$$\underset{\approx}{P}+\sum_{\mu}\underset{\approx}{P}_{\mu}=p\,\underset{\approx}{I}. \qquad (6)$$

If the effects of cloud and rainwater are neglected, the total pressure (P) in the system is due entirely to the partial pressures of air (P_d) and water vapour (e).

The assumptions (5) and (6) may be applied to (4), which, on application of the divergence theorem, yields

$$\frac{\partial}{\partial t}\left\{\rho\,\underset{\sim}{v}+\sum_{\mu}\rho_{\mu}\underset{\sim}{v}_{\mu}\right\}+\nabla\cdot\left\{\rho\,\underset{\sim}{v}\underset{\sim}{v}\right\}+\nabla\cdot\sum_{\mu}\rho_{\mu}\underset{\sim}{v}_{\mu}\underset{\sim}{v}_{\mu}$$

$$=-\nabla P-g\rho_{T}\hat{\underset{\sim}{k}}, \qquad (7)$$

since the volume τ over which the integration is performed is arbitrary. The summations (μ) in (7) are over all the water variables in the system ($\mu = v, c, r$ for vapour, cloud and rain). Using the definitions

$$x=\rho_{v}/\rho\ ,\quad c=\rho_{c}/\rho\ ,\quad r=\rho_{r}/\rho,$$

and the equations (1,3) it is possible to eliminate the time derivatives of density occurring in (7) to yield the momentum equation

$$\rho_T \frac{D\underset{\sim}{V}}{Dt} + r\rho\left(\frac{D\underset{\sim}{V}_T}{Dt} + \underset{\sim}{V}_T \cdot \nabla(\underset{\sim}{V}+\underset{\sim}{V}_T)\right) + \rho \underset{\sim}{V}_T S_r = -g\rho_T \underset{\sim}{\hat{k}} - \nabla p . \qquad (8)$$

Further progress with (8) cannot be made without specifying the form of $\underset{\sim}{V}_T$. Following Miller and Pearce (1974) the functional form of $\underset{\sim}{V}_T$ is taken as

$$\underset{\sim}{V}_T = -V_T \underset{\sim}{\hat{k}} = -21.18\, r^{.2}\, \underset{\sim}{\hat{k}} \quad (m\, s^{-1}). \qquad (9)$$

The pressure term $\rho^{-1}\nabla p$ can also be simplified by introducing the modified Exner function

$$\Pi = \left(\frac{p}{p_s}\right)^{R/c_p} = \left(\frac{p_d + e}{p_s}\right)^{R/c_p},$$

where p_s is a constant reference pressure. Defining a potential temperature θ as $\theta = T/\Pi$, and using the gas law $p_d = R\rho T$ for dry air, clearly

$$\begin{aligned}\frac{1}{\rho}\nabla p &= \frac{RT}{p_d}\nabla p = \frac{R\theta\Pi}{p_d}\nabla p \\ &= \frac{p}{p_d}\cdot\frac{R\theta\Pi}{p}\nabla p \\ &= (1+e/p_d)\, c_p \theta \nabla \Pi .\end{aligned}$$

But the humidity mixing ratio x is given by

$$x = .622\, e/p_d,$$

so that

$$\frac{1}{\rho}\nabla p = (1+1.61x)\, c_p \theta \nabla \Pi . \qquad (10)$$

In the horizontal equations of motion, the pressure term $\rho_T^{-1}\nabla_H p$ is approximated as $c_p\theta\nabla_H\Pi$, so that terms in x, c and r are neglected. For the vertical pressure gradient, first order terms in x, c and r must be retained if the effects of the water variables on bouyancy are to be properly modelled. It follows that

$$\frac{1}{\rho(1+x+c+r)}\frac{\partial p}{\partial z} = \frac{1+1.61x}{(1+x+c+r)}\cdot c_p\theta\frac{\partial \Pi}{\partial z},$$

from (10), and is approximately equal to

$$c_p\theta\frac{\partial \Pi}{\partial z} + (.61x - c - r)\, c_p\theta\frac{\partial \Pi}{\partial z},$$

or

$$\frac{1}{\rho_T}\cdot\frac{\partial p}{\partial z} \sim c_p\theta\frac{\partial \Pi}{\partial z} - g(.61x - c - r). \qquad (11)$$

In (11) the hydrostatic approximation has been made on the pressure gradient multiplying the small terms on the right. Using (9), (10) and (11) in the momentum

equation (8) yields the component equations:

$$\left.\begin{aligned}
\frac{Du}{Dt} - r V_T \frac{\partial u}{\partial z} &= -C_p \theta \frac{\partial P}{\partial x} \\
\frac{Dv}{Dt} - r V_T \frac{\partial v}{\partial z} &= -C_p \theta \frac{\partial I}{\partial y} \\
\frac{Dw}{Dt} - r V_T \frac{\partial w}{\partial z} + r\left\{V_T \frac{\partial V_T}{\partial z} - \frac{D V_T}{Dt} - \frac{V_T S_r}{r}\right\} &= -g(1 - .61x + C + r) - C_p \theta \frac{\partial I}{\partial z}.
\end{aligned}\right\} \quad (12a\text{-}c)$$

From equation (12c) it is possible to see the effects of the water variables on the bouyancy within the system. Thus it is increased with the presence of water vapour (x) and decreased due to the liquid loading of cloud and rain.

Finally a prognostic equation for the variable P must be derived. Using the definitions $P = \left(\frac{p_d + e}{p_s}\right)^{R/C_p}$, $p_d = R\rho T$ and the continuity equation for dry air there results the equation

$$\frac{DP}{Dt} + (\gamma - 1) P \nabla \cdot \underset{\sim}{V} = \frac{(\gamma - 1) P}{\theta}\left\{\frac{D\theta}{Dt} + 1.61\theta \frac{Dx}{Dt}\right\}, \quad (13)$$

where terms of order x^2 and higher have been neglected.

THERMAL ENERGY EQUATION

An equation for the thermal energy of the system as a whole can be derived by writing down the enthalpy (H). By definition

$$H = C_p T + x C_p' T + \ell C_1 T + x L(T) \quad (14)$$

where C_p is the usual specific heat of air at constant pressure. C_p' and C_1 are the specific heats at constant pressure of water vapour and liquid respectively. ℓ is the liquid water content ($= r + c$) and the latent heat of evaporation (L) as a function of temperature (T in °C) is

$$L(T) = (2.5099 - .00234T) \times 10^6 \text{ J Kg}^{-1} \quad (15)$$

To a good approximation, the variation of L with temperature may be neglected along with the terms $x C_p' T$ and $\ell C_1 T$ in (14). Hence from equation (14):

$$\frac{DH}{Dt} = C_p \frac{DT}{Dt} + L \frac{Dx}{Dt}$$
$$= \frac{D\Phi}{Dt} + \frac{1}{\rho} \frac{DP}{Dt}.$$

Since the temperature T is given by $T = \theta P$ and

$$\frac{1}{\rho} \frac{DP}{Dt} \sim C_p \theta \frac{DP}{Dt} \quad (\text{cf eqn. (10)}),$$

then the thermal energy equation may be written in the form

$$\frac{D\theta}{Dt} = \frac{\theta}{TC_p}\left\{\frac{DQ}{Dt} - L\frac{Dx}{Dt}\right\}. \qquad (16)$$

In (16), $\frac{DQ}{Dt}$ defines a heat source used to initiate convection in the model.

Equations (12a-c), (13) and (16) are the final results of this section and describe the overall dynamics of the system considered here.

§ 3 CLOUD PHYSICS

The cloud physics is modelled by defining the source terms $S_\mu (\mu = V, C, r)$ and the rainwater terminal velocity $\underset{\sim}{V}_T$. Following a scheme similar to that of Liu and Orville (1969) and Miller and Pearce (1974) the rainwater source term is broken down into three processes:

$$S_r = P_1 + P_2 + P_3.$$

The evaporation of raindrops (P_1) is taken as

$$P_1 = \beta(x - x_{sat}) \quad Kg.Kg^{-1}.s^{-1}, \qquad (17)$$

where x_{sat} is the saturated humidity mixing ratio. β is a constant typically of order $10^{-3}\,s^{-1}$. The accretion of cloud by rain (P_2) is parameterised as

$$P_2 = \gamma C r^{.95} \times 707.95 \quad Kg.Kg^{-1}.s^{-1}, \qquad (18)$$

where γ is typically of order $10^{-3}\,s^{-1}$. The constant (707.95) appearing in (18) takes into account the fact that the units used are M.K.S. throughout and not the mixed units gms/kg/s as in Miller and Pearce (1974). Finally the autoconversion of cloud to rain (P_3) is defined as

$$P_3 = \alpha(C - C_{crit}) \quad Kg.Kg^{-1}.s^{-1}, \qquad (19)$$

where again α is of order $10^{-3}\,s^{-1}$. C_{crit} is a cloud water value below which there is no conversion.

With the above definition for S_r, it is possible to write down the remaining source terms S_v and S_c as

$$S_c = -P_2 - P_3 + S_{c/e}$$
$$S_v = -P_1 - S_{c/e}$$

Note that because of the form of P_1 (17) it is essentially a negative quantity. The contribution to the source terms S_c and S_v due to condensation/evaporation between cloud and water is denoted by $S_{c/e}$. $S_{c/e}$ is calculated by an adjustment process on the assumption that the air never becomes supersaturated. Increments

$\Delta\theta_1$ and Δx_1 are first calculated using the advection equations

$$\frac{D\theta}{Dt} = 0 \quad , \quad \frac{Dx}{Dt} = 0 .$$

Putting $\theta_1 = \theta + \Delta\theta_1$ (hence T_1) and $x_1 = x + \Delta x_1$, if $x_1 \leq x_{sat}(T_1)$ and no cloud water is present, no further adjustment is made, so that

$$S_{c/e} = 0$$
$$\Delta\theta = \Delta\theta_1 \ \& \ \Delta x = \Delta x_1 .$$

If $x_1 > x_{sat}(T_1)$, a correction (ΔT) to ΔT_1 is made using the truncated Taylor expression:

$$x_1 - \left(x_{sat}(T_1) + \Delta T\, x'_{sat}(T_1) \right) = \frac{c_p \Delta T}{L} ,$$

where $'$ denotes $\frac{\partial}{\partial T_1}$ and

$$x_{sat} = \cdot 622 \frac{e_{sat}}{P_d} .$$

Clearly in this case the excess water vapour which would otherwise make the air supersaturated is condensed to form cloud and the latent heat released is accounted for in the thermal energy equation. From an approximate form of Clapeyron's equation x'_{sat} is given by

$$x'_{sat} = \frac{x_{sat} L}{R_v T^2} ,$$

where R_v is the gas constant for water vapour. A similar procedure is carried out for the evaporation of cloud water when $x_1 < x_{sat}(T_1)$. Figure 1 illustrates the various physical processes occurring between the water variables in the model.

The fall velocity of raindrops ($\underset{\sim}{V}_T$), relative to the other components, is parameterised as a function of the rainwater concentration term r:

$$\underset{\sim}{V}_T = -V_T \underset{\sim}{\hat{k}} ,$$

where

$$V_T = 21.18\, r^{\cdot 2} \quad m\, s^{-1} .$$

§ 4 SOME NUMERICAL ASPECTS OF THE MODEL

The basic formulation of the model follows that of Tapp and White (1976) with a non-hydrostatic comprehensible model in which the sound wave terms are treated implicitly. The grid used is cartesian and the current integration domain is an array of 24 x 24 points with a horizontal grid length of 1 Km and 19 unevenly spaced levels extending to 18 Km. (See figure 3 for vertical grid structure.) The equations (12a-c), (13), (16) plus the three continuity equations for the water

variables are integrated in time (timestep = 10s) using a second order semi-implicit centred scheme (leap frog). The space differencing is also centred. The numerical scheme (for the above grid configuration) requires the solution to eighteen Helmholtz equations every timestep. This is accomplished using a direct method described by Tapp (1976). As discovered by Tapp (1977) direct methods afford other programming advantages (particularly core store requirements) and these have been incorporated into the cumulus model. In principle there is now no restriction on the number of rows in the grid, although in practice this will be governed by CPU requirements. The special staggered grid arrangement between even and odd timesteps is given in Tapp (1977).

The variables Π and θ are written in terms of deviations Π_1 and θ_1 from a steady neutrally stable state of the atmosphere $\Pi_0(z)$ and θ_0 = constant. For the cumulus model with an atmosphere extending to above the tropopause this is not a very representative profile to choose. Although to date no computational problems have arisen, it would still be desirable to choose a more realistic base $\theta_0 = \theta_0(z)$.

In performing accurate thermodynamic computations in height co-ordinate convection models both temperature and pressure variations can occur at each gridpoint. To compute the saturation vapour pressure (a function of temperature) requires both θ and the complete pressure field Π. Since Π is computed from equation (13) this would involve, in principle, introducing implicit relations requiring iterative solution. In a pressure co-ordinate system, by definition, pressure is constant at a grid point. Wilhelmson and Ogura (1972) suggest that, in general, the pressure perturbations associated with convection are rather small; this is also supported by Miller and Pearce (1974). The pressure variations found occurring in the model described here show only very small fluctuations; typically $\delta\Pi \sim 10^{-6}$ to 10^{-5} (equivalent to 10^{-3} to 10^{-2} mb.) per timestep.

The equations of motion (12a-c) have rigorously included the effects of the fall velocity of rainwater ($\underset{\sim}{V}_T$). In fact some numerical results indicate that this effect is small in the equations of motion and can be neglected without significant error; so that in (12a-c) $\underset{\sim}{V}_T$ can be effectively equated to zero. Of course the effect of the fall velocity on rainwater itself cannot be neglected and

must be retained in the appropriate continuity equation.

When first running trial integrations with the model problems arose over the standard finite difference approximation to the water variable advection equations for cloud and rain. Since these two quantities tend to be rather discontinuous in space, negative values quickly accumulated in the model once cloud and rain formed. To circumvent this problem a simple quasi-Lagrangian scheme has been developed (see for example Haltiner 1971, p. 208) and appears to give satisfactory results. This scheme is first order in time and first order accurate in space (see appendix 2).

To prevent non-linear instability occurring in the model diffusion terms are added to the predictive equations for velocity $\underset{\sim}{v}$, potential temperature θ and humidity χ. The form currently in use differentiates between the horizontal and vertical directions and takes the form (for variable ψ):

$$K_H \nabla_H^2 \psi + K_z \frac{\partial^2 \psi}{\partial z^2},$$

where

$$K_H = K_H' \left| \nabla_H^2 \psi \right| \quad \text{and} \quad K_z = K_z' \left| \frac{\partial^2 \psi}{\partial z^2} \right|.$$

K_H' and K_z' are defined in terms of the horizontal and vertical grid lengths Δx and Δz respectively, and are given by

$$K_H' = \zeta (\Delta x)^3 , \quad K_z' = \zeta (\Delta z)^3.$$

ζ is a constant which for the velocity fields has a value of .075, for the potential temperature field a value .05, and for the humidity field a value 12.5. These ζ- values give sufficient diffusion to control any small scale roughnesses that occur but do not appear to inhibit development of the main features. Further refinement of the values may however prove necessary.

To initiate convection in the model, simple fluxes of heat and moisture are applied at the lowest two levels (300 m and 700 m) over four adjacent gridpoints at the centre of the area. The forcing is applied gradually during the beginning of an integration. Currently the model is being run with the forcing amplitudes of heat and moisture taken as $2°C$ and $1 gm\ kg^{-1}$ respectively. Both the amplitudes and period of forcing can be varied as desired. In § 5 some results of an integration are discussed, in which the period of initiation is taken as $\frac{3}{4}$ hr.. The cloud physics parameters used in this run are

$$\alpha = \beta = \gamma = 10^{-3}\ s^{-1},$$

and the critical cloud content

$$C_{crit} = 5\times10^{-4}\ Kg\ Kg^{-1}.$$

A simple surface drag term is included in the horizontal equations of motion (for the lowest level) of the form

$$-\rho^{-1}\frac{\partial \underset{\sim}{\tau}}{\partial z}$$

Where the stress $\underset{\sim}{\tau}$ is given by

$$\underset{\sim}{\tau} = \rho C_D |\underset{\sim}{V}_H| \underset{\sim}{V}_H$$

and C_D is the drag coefficient taken as 5×10^{-3}.

§5 INITIAL AND BOUNDARY CONDITIONS

The model is being verified against the sounding used in the Met O 15 model, subject to interpolation differences between the co-ordinate systems. The initial dew point and temperature distributions are shown in figure 2. The atmosphere is assumed initially to be at rest in a state of hydrostatic balance everywhere, so that from equation (12c)

$$C_p\theta\frac{\partial \Pi}{\partial z} + g(1-\cdot61x) = 0.$$

So far the model has only been run on a large enough grid to allow the normal velocity at the boundary (including the top) to be specified as zero throughout the integration. For computational reasons zero normal velocity is still considered as outflow from the region so that boundary values are obtained from information within the grid area.

(i) At outflow, tendencies of the potential temperature are calculated by upstream advection, while the tangential wind components and water variables for time level $n+1$ are computed using the simple condition

$$\phi^{n+1}_{G} = \phi^{n}_{G-1}$$

$G-1$ is the first internal grid point in the model.

(ii) The pressure round the boundaries is determined from the normal pressure gradients calculated from the equations of motion perpendicular to the boundaries. This leads to Neumann (ie normal gradient) type boundary conditions for the Helmholtz equations.

(iii) The horizontal and vertical diffusion is increased by a factor of four for the three grid points nearest the boundary.

AN EXAMPLE INTEGRATION

In this run the forcing terms were applied for a period of three quarters of an hour. Figures 4a-6c show cross-sections for wind vectors, cloud and rainwater. By 35 min convection is well developed with updraft speeds of about 6,5 $m\,s^{-1}$ (figure 4a). Cloud extends from about 1400 m to 3000 m at this stage (figure 4b) with maximum values of about 2 gms/kg. Rainwater has formed with values of 1 gm/kg (figure 4c) and extends over a depth of about a kilometre from 2200 m downwards. At 45 min fairly rapid developments have taken place under the influence of the liquid water loading and rainwater evaporation. The cloud base (figure 5b) has lifted to about 3300 m and a downdraft (maximum value 4 m/s) has formed down to the surface (figure 5a). A smaller compensating updraft can be seen further out either side of the downdraft. Rain (figure 5c) extends down to about the 3rd model level - presumable rainwater evaporation is sufficient to prevent any rain reaching the surface. The heating by this stage is switched off. By 55 min updraft speeds in the main convective element have reduced to 5 $m\,s^{-1}$ and the cloud base (figure 6b) has lifted to about 5200 m with rain extending down to only the 6th model level. The main dowdraft (figure 6a) has weakened (to about 3 m/s) and extends to only a few levels below the cloud base, while a weaker updraft (2 m/s) has formed pushing the descending air outwards from the main downdraft core.

§ 6 CONCLUSIONS

So far the model is being verified against the sounding used in the Met O 15 model. It is hoped to run the model to simulate convective growth in supercell storms (see for example Browning and Foote 1976). Since the computer program has been written to encompass a wide variety of grid configurations, it should provide a powerful research tool in convective modelling. Indeed, with the inclusion of a detailed boundary layer scheme, the model could be used to study the effects of cumulus dynamics on sea-breeze situations in special cases; or the influence of the structure of the surface layer on the growth and intensity of convection. In this way the model has the potential for constructing valuable solutions to important problems in mesoscale meteorology.

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APPENDIX 1

NOMENCLATURE AND DEFINITIONS

All units used throughout this technical note are M.K.S.

ρ	density of dry air
ρ_v	density of water vapour (v)
ρ_c	density of cloud water (c)
ρ_r	density of rainwater (r)
ρ_T	total density equal to $\rho + \rho_v + \rho_c + \rho_r$
x	humidity mixing ratio ρ_v/ρ
c	cloud water mixing ratio ρ_c/ρ
r	rainwater mixing ratio ρ_r/ρ
α, β, γ	cloud physics parameters
$\underset{\sim}{V}$	velocity of air with cartesian components (U, V, W)
$\underset{\sim}{V}_T$	terminal full velocity of rainwater
$\underset{\sim}{V}\underset{\sim}{V}$	denotes tensor $\{V_i V_j\}$ $(i, j = 1, 2, 3)$
S_μ	source/sink term for quantity μ $(\mu = v, c, r)$
$\frac{D}{Dt}$	Eulerian derivative $\frac{\partial}{\partial t} + \underset{\sim}{V}\cdot\nabla$
P_d	pressure of dry air
e	pressure of water vapour $1{\cdot}61 \times P_d$

APPENDIX 2

A QUASI-LAGRANGRIAN SCHEME

For the advection of cloud and rain in the model a quasi-Lagrangian scheme has been developed (see for example Haltiner 1971, p. 208). The essence of the approach is to determine at time t the position of the particle which will arrive at a particular grid point at the future time $t+\Delta t$. Consider the simple one-dimensional advection equation

$$\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial x} = 0 \quad , \quad U \text{ CONSTANT.} \qquad \text{A1}$$

The analytic solution is simply $F(x-Ut)$. Hence the value of F at a future time $(n+1)\delta t$ AT the point x is given by the value of F at the point $x-U\delta t$. Thus if

$$F(x,t) = F(m\delta x, n\delta t) = F^{n}_{m},$$

then

$$F(m\delta x,(n+1)\delta t) = F(m\delta x - U\delta t, n\delta t).$$

This is illustrated in figure 4. Next the value of F at the point $m\delta x - U\delta t$ may be obtained by interpolation as

$$F(m\delta x - U\delta t, n\delta t) = \frac{U\delta t}{\delta x} F^{n}_{m-1} + F^{n}_{m}\left(1 - \frac{U\delta t}{\delta x}\right)$$

or

$$F^{n+1}_{m} = F^{n}_{m} - \frac{U\delta t}{\delta x}\left(F^{n}_{m} - F^{n}_{m-1}\right) \qquad \text{A2}$$

Equation A2 may be regarded as a truncated Taylor series with the derivatives approximated by a one sided difference. In fact A2 is simply the upstream differencing approximation to the advection equation A1. The process of linear interpolation may be extended in a straightforward manner to three dimensions. The scheme may be extended to give second order accurate spatial interpolation.

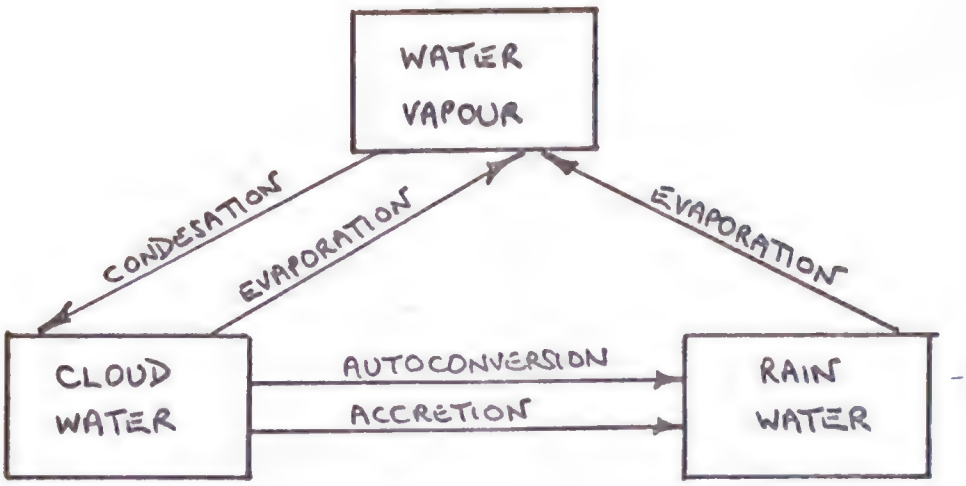

FIGURE 1. BLOCK DIAGRAM OF MICROPHYSICS PROCESSES.

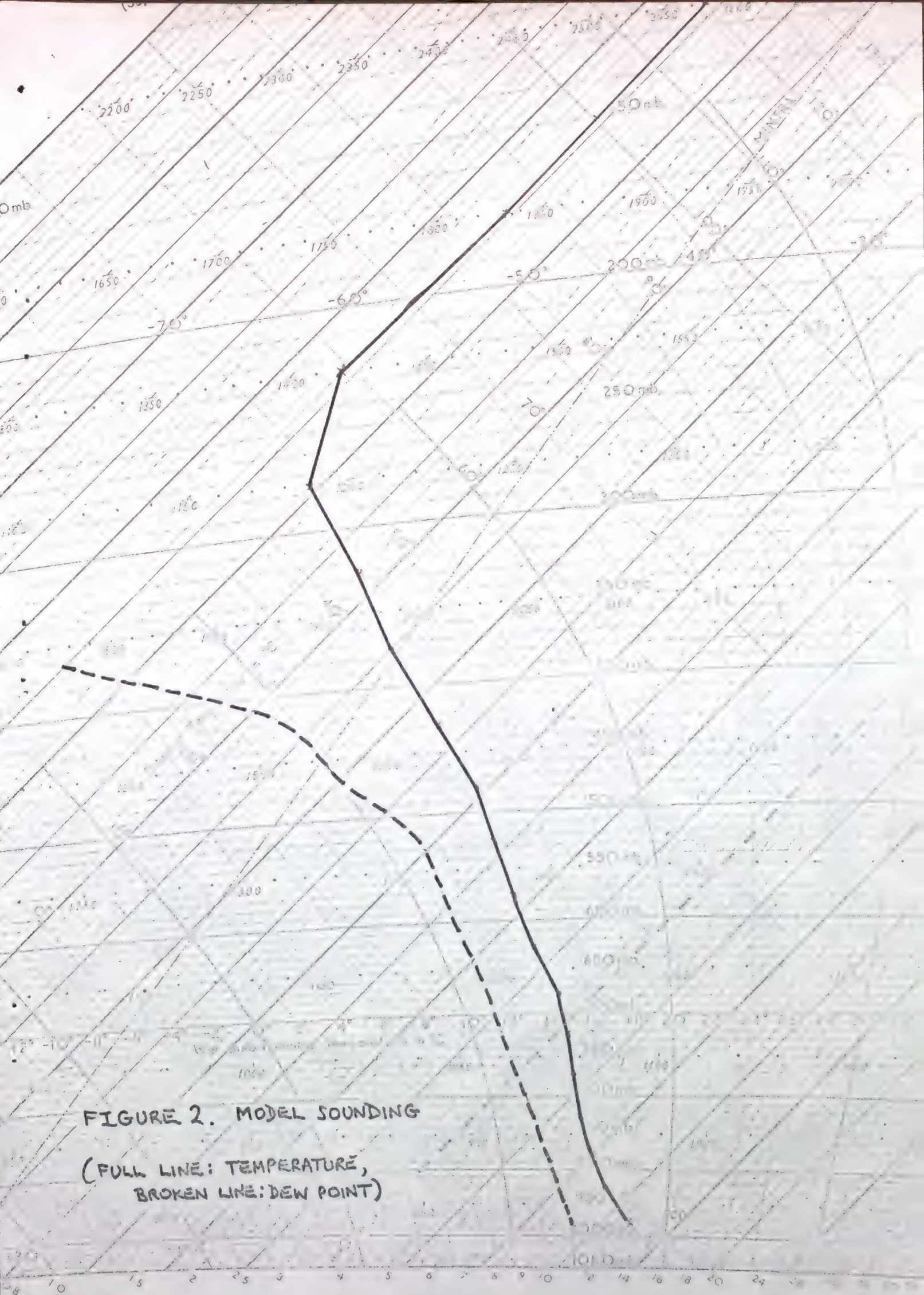

FIGURE 2. MODEL SOUNDING

(FULL LINE: TEMPERATURE,
BROKEN LINE: DEW POINT)

FIGURE 3. DISTRIBUTION OF LEVELS IN THE CUMULUS MODEL.

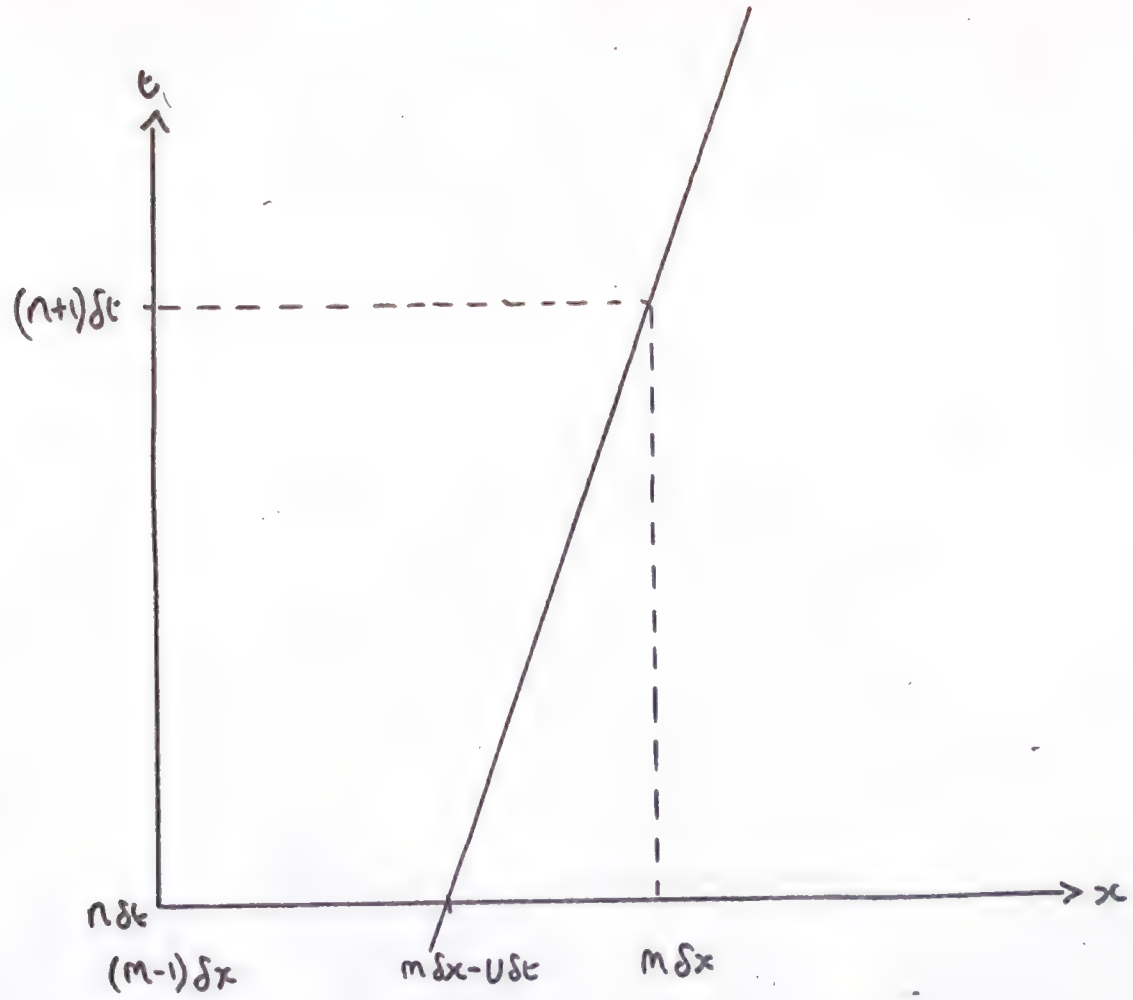

FIGURE 4. A QUASI - LAGRANGIAN SCHEME

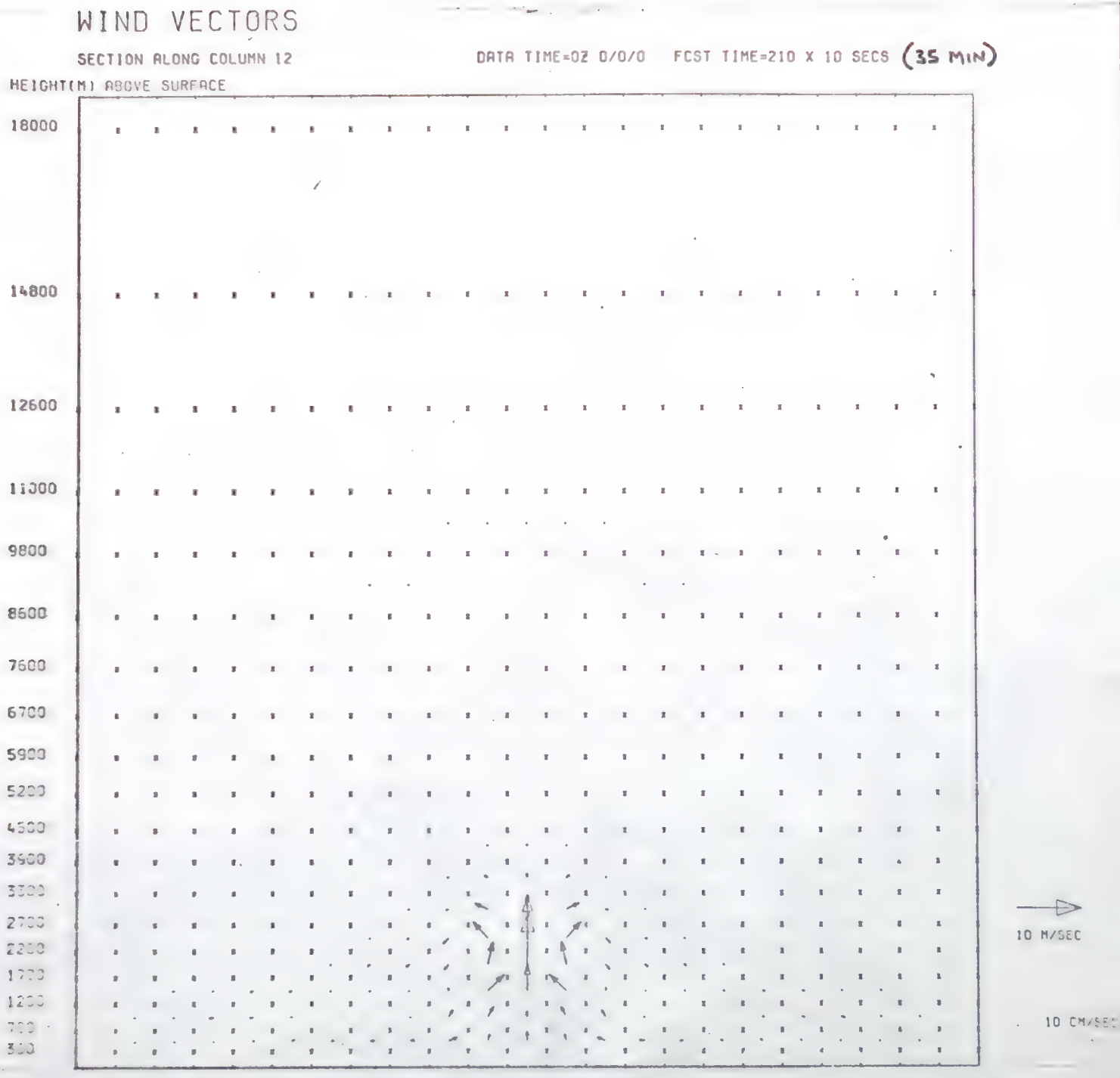
WIND VECTORS
SECTION ALONG COLUMN 12
DATA TIME=02 0/0/0 FCST TIME=210 X 10 SECS (35 MIN)
HEIGHT(M) ABOVE SURFACE
18000
14800
12600
11300
9800
8600
7600
6700
5900
5200
4500
3900
3300
2700
2200
1700
1200
700
300
10 M/SEC
10 CM/SEC

FIGURE 4a.

CLOUD WATER ISOPLETH INTERVAL = 1 GMS/KG

SECTION ALONG COLUMN 12 DATA TIME=02 0/0/0 FCST TIME=210 X 10 SECS (35 MIN)

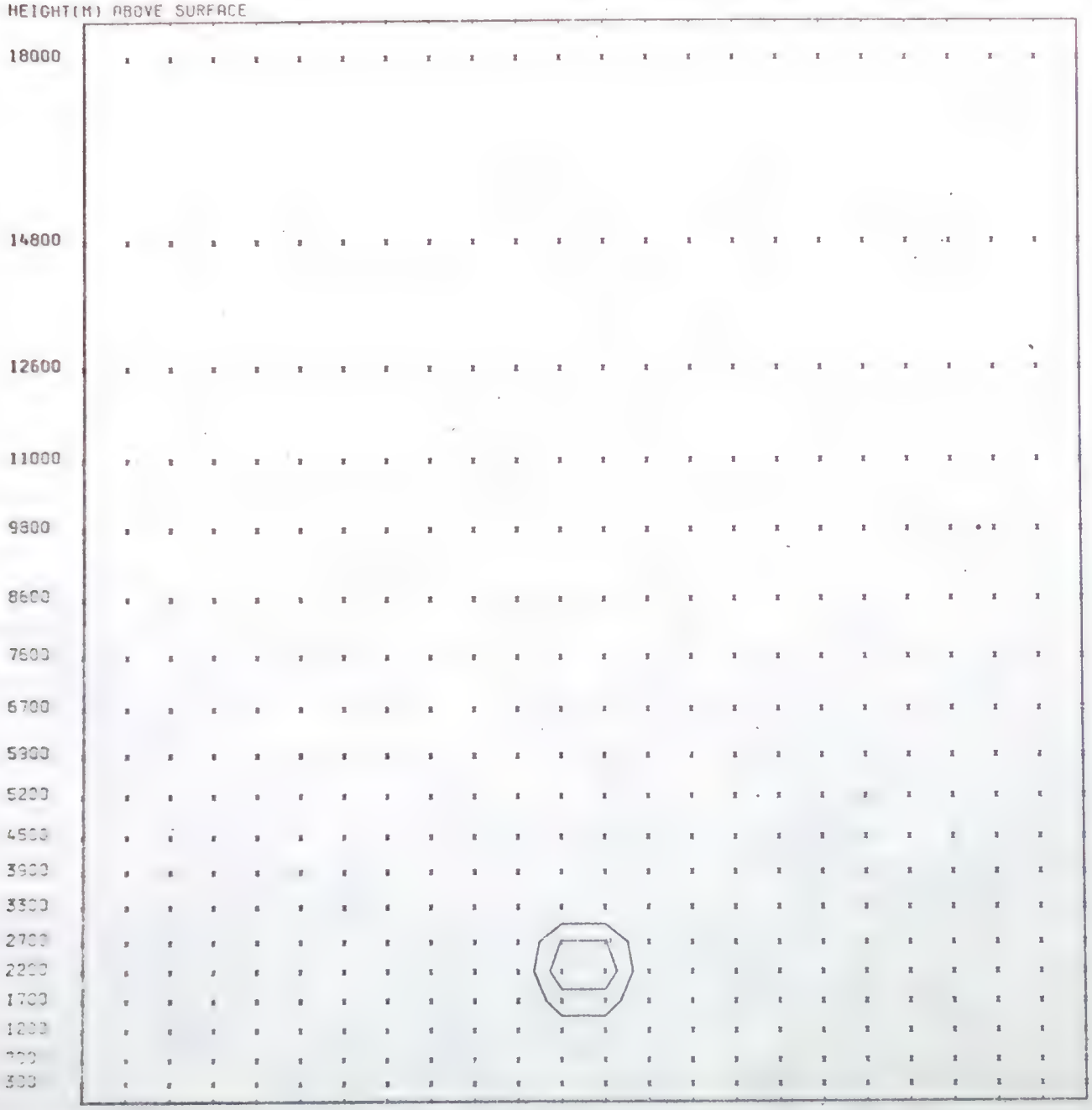

FIGURE 4G.

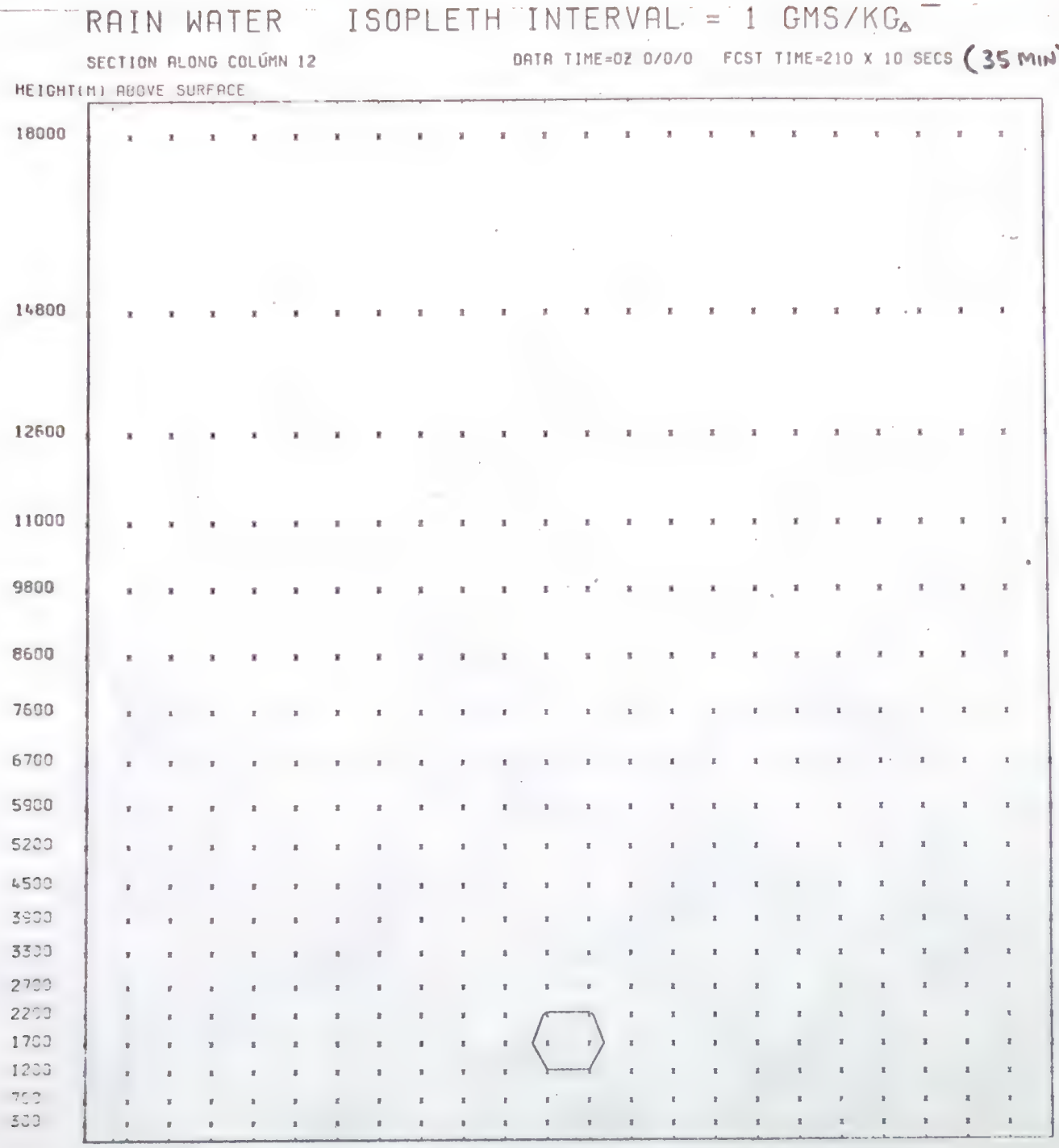
RAIN WATER ISOPLETH INTERVAL = 1 GMS/KG
SECTION ALONG COLUMN 12
DATA TIME=02 0/0/0 FCST TIME=210 X 10 SECS (35 MIN)
HEIGHT(M) ABOVE SURFACE
18000
14800
12600
11000
9800
8600
7600
6700
5900
5200
4500
3900
3300
2700
2200
1700
1200
700
300

FIGURE 4c.

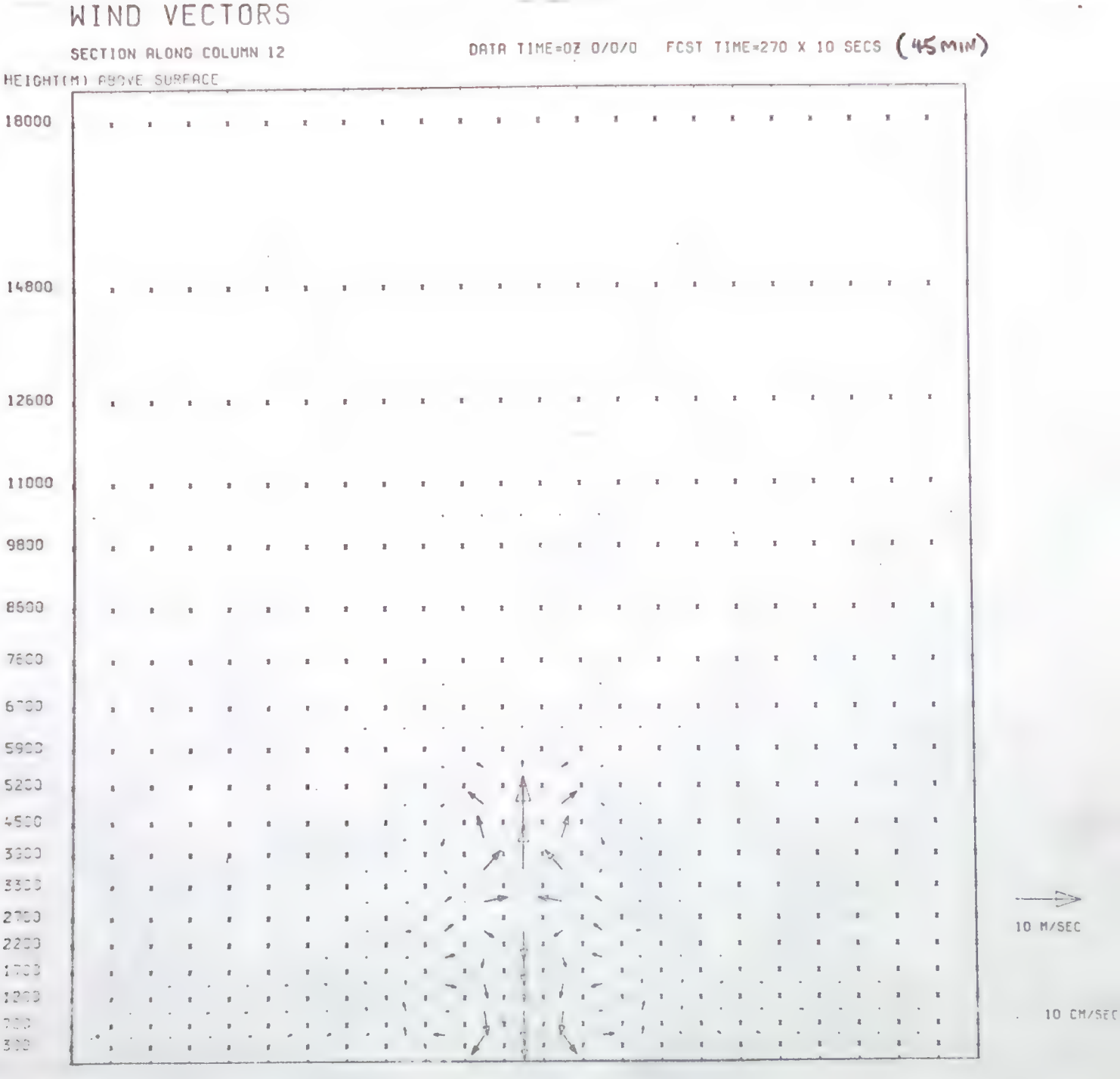
WIND VECTORS
SECTION ALONG COLUMN 12
DATA TIME=0Z 0/0/0 FCST TIME=270 X 10 SECS (45 MIN)
HEIGHT(M) ABOVE SURFACE
18000
14800
12600
11000
9800
8600
7600
6700
5900
5200
4500
3800
3300
2700
2200
1700
1200
700
300
10 M/SEC
10 CM/SEC

FIGURE 5a.

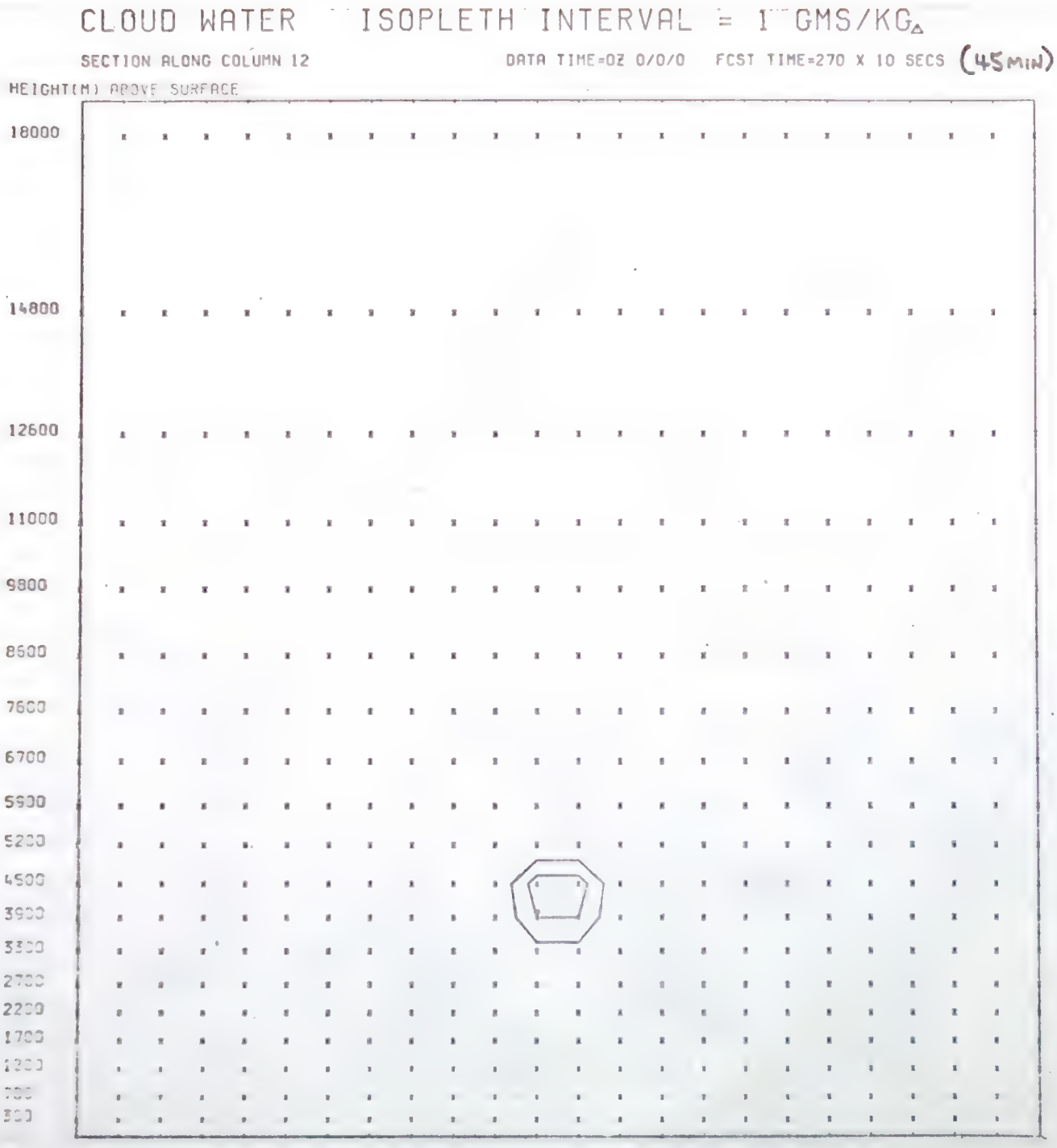
CLOUD WATER ISOPLETH INTERVAL = 1 GMS/KG
SECTION ALONG COLUMN 12
DATA TIME=0Z 0/0/0 FCST TIME=270 X 10 SECS (45 MIN)
HEIGHT(M) ABOVE SURFACE
18000
14800
12600
11000
9800
8600
7600
6700
5900
5200
4500
3900
3300
2700
2200
1700
1200
700
300

FIGURE 5G.

RAIN WATER ISOPLETH INTERVAL = 1 GMS/KG$_{\Delta}$

SECTION ALONG COLUMN 12 DATA TIME=0Z 0/0/0 FCST TIME=270 X 10 SECS (45MIN)

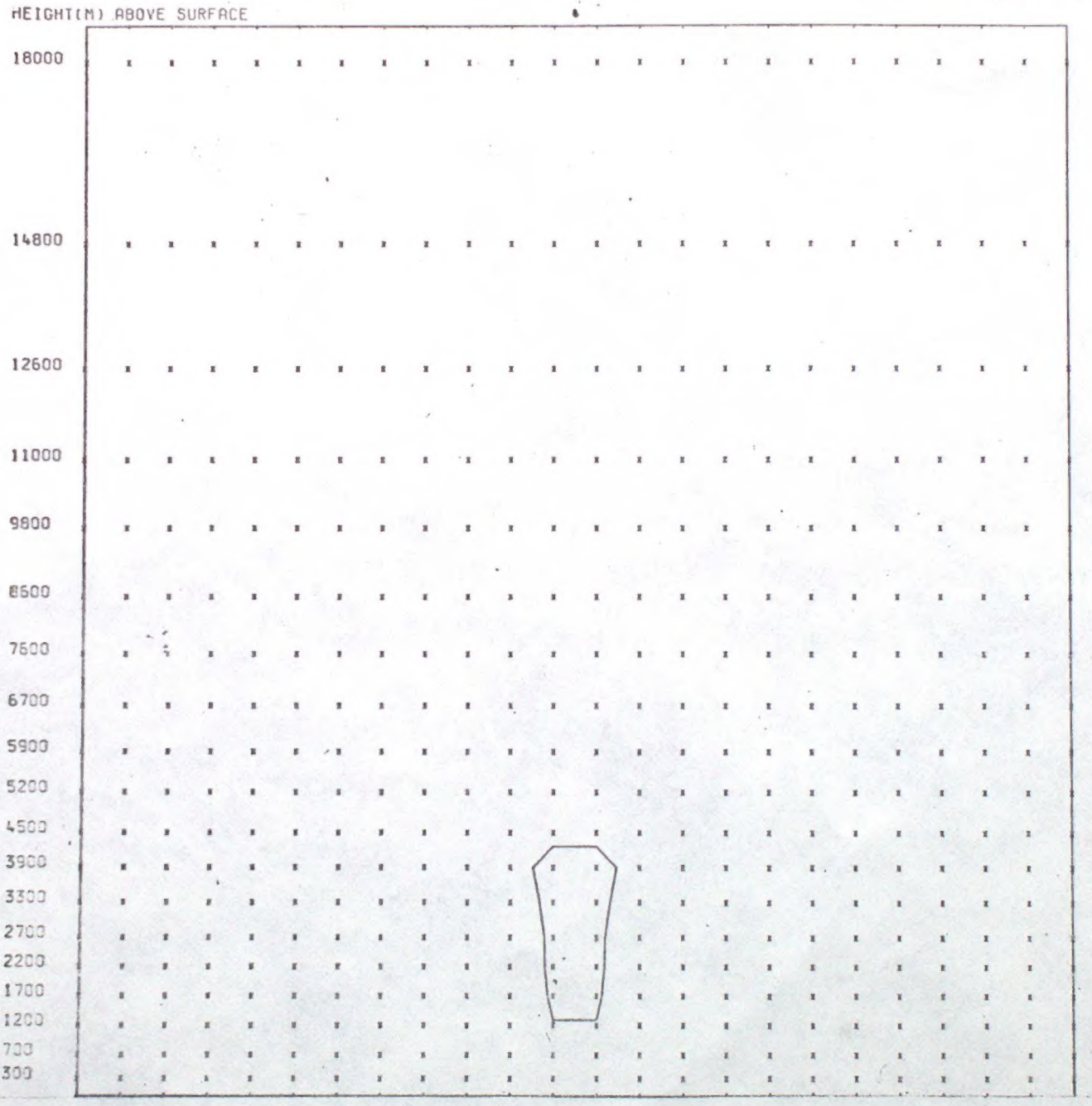

FIGURE 5c.

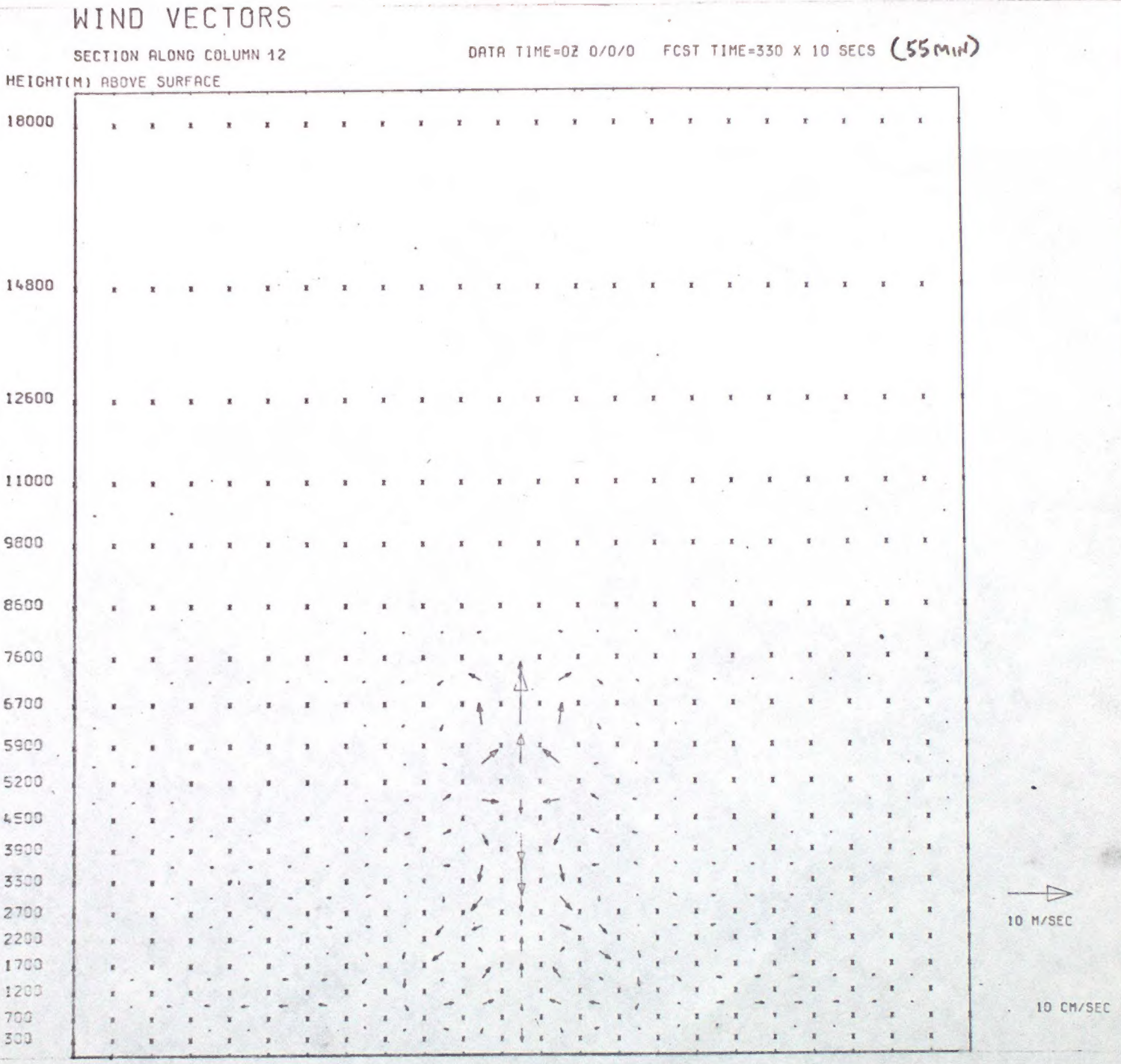
WIND VECTORS
SECTION ALONG COLUMN 12
DATA TIME=0Z 0/0/0 FCST TIME=330 X 10 SECS (55 min)
HEIGHT(M) ABOVE SURFACE
18000
14800
12600
11000
9800
8600
7600
6700
5900
5200
4500
3900
3300
2700
2200
1700
1200
700
300
10 M/SEC
10 CM/SEC

FIGURE 6a.

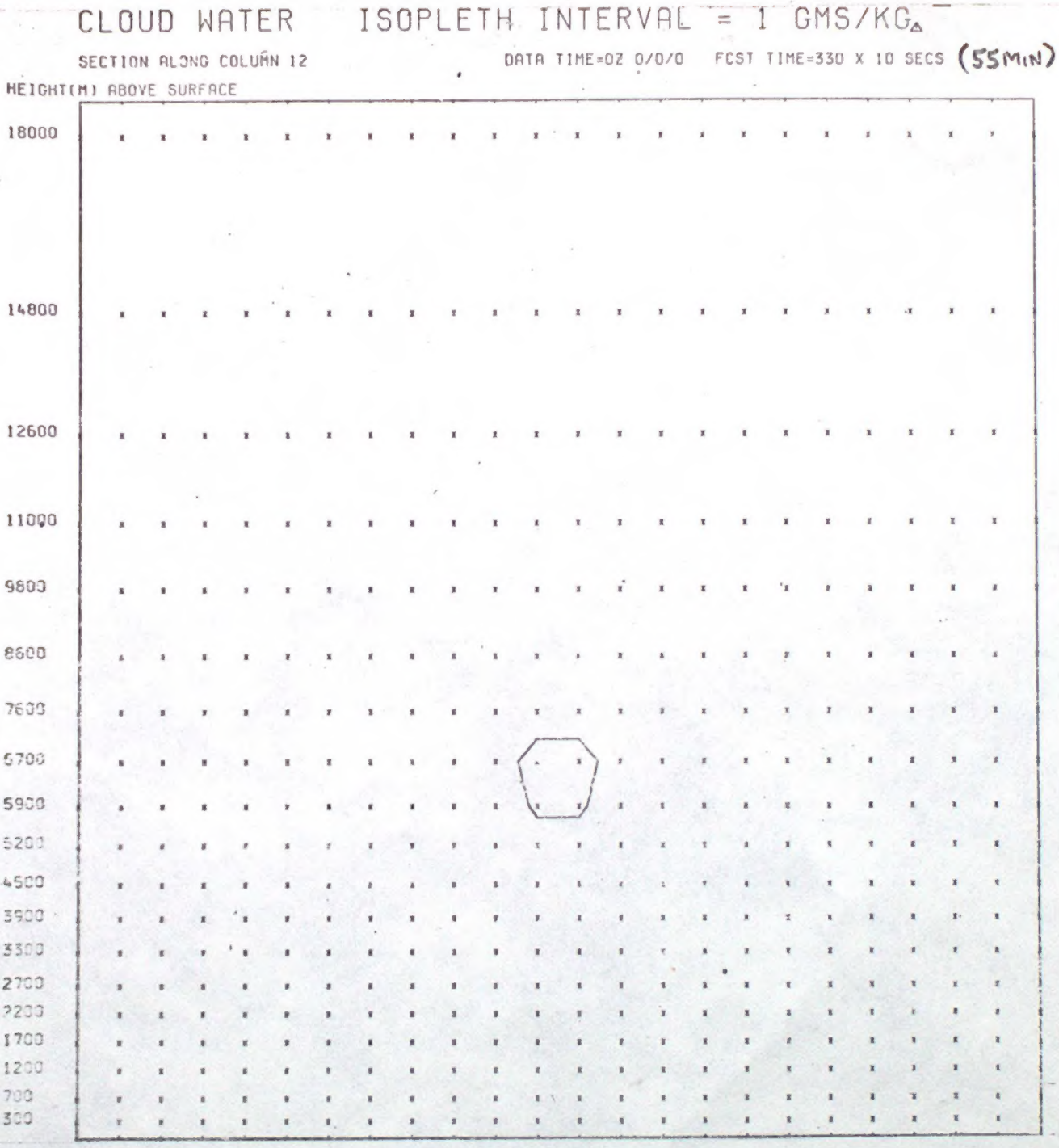
CLOUD WATER ISOPLETH INTERVAL = 1 GMS/KG
SECTION ALONG COLUMN 12
DATA TIME=02 0/0/0 FCST TIME=330 X 10 SECS (55MIN)
HEIGHT(M) ABOVE SURFACE
18000
14800
12600
11000
9800
8600
7600
6700
5900
5200
4500
3900
3300
2700
2200
1700
1200
700
300

FIGURE 66.

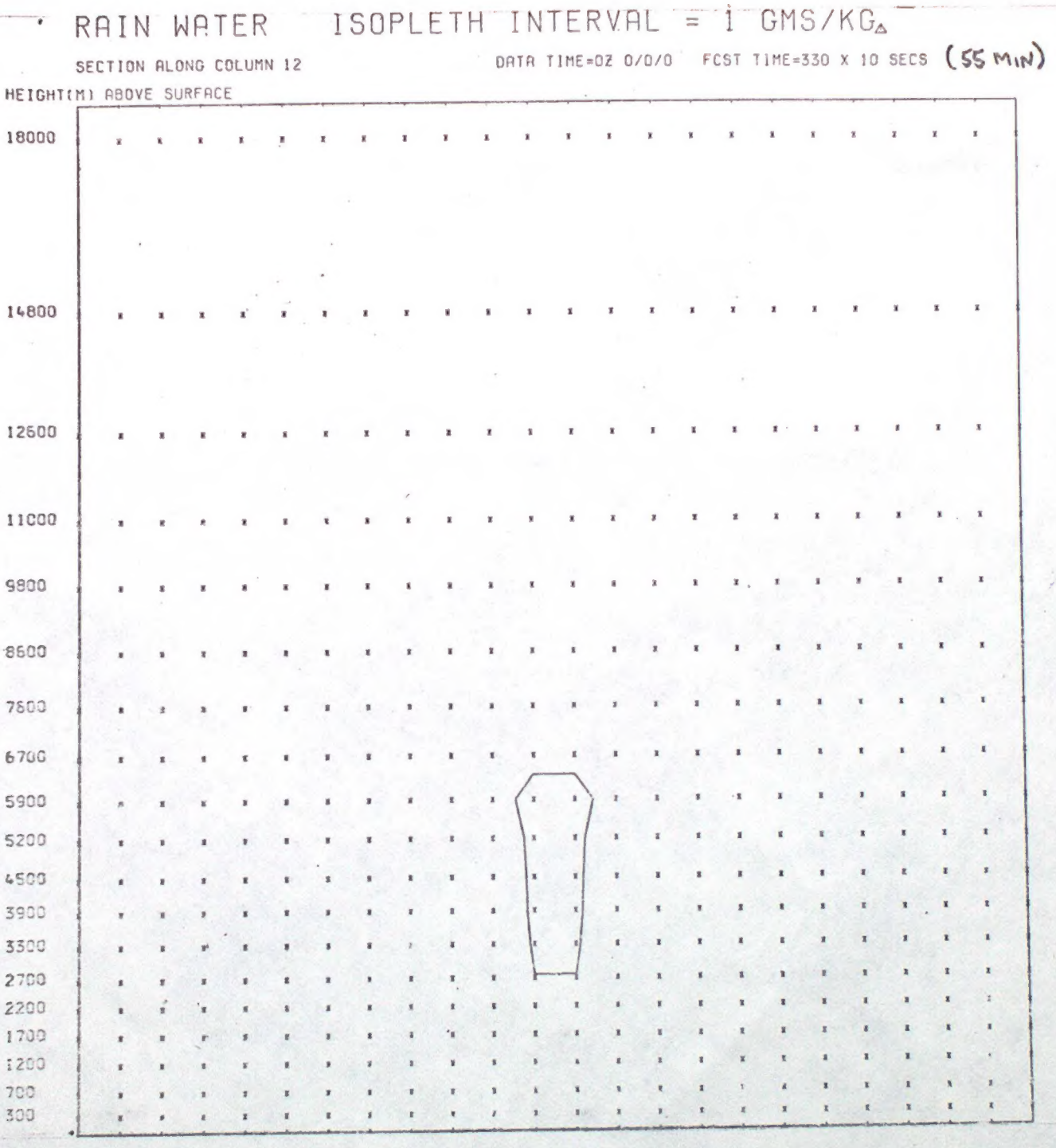
RAIN WATER ISOPLETH INTERVAL = 1 GMS/KG
SECTION ALONG COLUMN 12
DATA TIME=02 0/0/0 FCST TIME=330 X 10 SECS (55 MIN)
HEIGHT(M) ABOVE SURFACE
18000
14800
12600
11000
9800
8600
7600
6700
5900
5200
4500
3900
3300
2700
2200
1700
1200
700
300

FIGURE 6c.